arXiv:astro-ph/9612230v1 25 Dec 1996

Single Close Encounters Do Not Make Eccentric Planetary Orbits

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Abstract

The recent discovery of a planet in an orbit with eccentricity $e = 0.63 \pm 0.08$ around the Solar-type star 16 Cyg B, together with earlier discoveries of other planets in orbits of significant eccentricity, raises the question of the origin of these orbits, so unlike the nearly circular orbits of our Solar system. In this paper I consider close encounters between two planets, each initially in a nearly circular orbit (but with sufficient eccentricity to permit the encounter). Such encounters are described by a two-body approximation, in which the effect of the attracting star is neglected, and by the approximation that their separation vector follows a nearly parabolic path. A single encounter cannot produce the present state of these systems, in which one planet is in an eccentric orbit and the other has apparently been lost. Even if the requirement that the second planet be lost is dropped, nearly circular orbits cannot scatter into eccentric ones.

1. Introduction

The recent discovery (Cochran, *et al.* 1996) of a planet orbiting 16 Cyg B with an eccentricity $e = 0.63 \pm 0.08$, combined with the earlier discoveries of a planet orbiting 70 Vir with $e = 0.40$ (Marcy & Butler 1996) and of one orbiting HD 114762 with $e = 0.35$ (Mazeh, Latham & Stefanik 1996), raise the question of how these eccentricities are produced. It is conventional to assume that planets form from a disc of gas and particles whose orbits have been circularized by frictional (viscous) dissipation. This is supported by the fact that the observed eccentricities of planets in the Solar System are generally quite small; even their finite but small eccentricities are probably explicable as the result of mutual gravitational perturbations. There are two apparent exceptions to this rule of small eccentricities in the Solar System: Pluto, with $e = 0.250$, resembles a strayed asteroid more than a planet; its present orbit is strongly affected by its 3:2 orbital resonance with Neptune, and its previous history may reflect a close encounter with Neptune (and Triton). Mercury has $e = 0.206$, which is less obviously explicable.

One possible explanation for large planetary eccentricities is gravitational interaction between two planets initially in nearly circular orbits, one of which was left in the observed eccentric orbit and the other expelled. Expulsion (either to infinity or into a very long period nearly parabolic orbit) is required because the data exclude a second planet with mass and period comparable to those of the observed planet. Conservation of energy and angular momentum permit a general constraint to be placed on the cumulative results of small perturbations by distant bodies over a long period of time, or of multiple close encounters, but this constraint is not powerful enough to be useful.

It is, however, possible to draw useful conclusions about the results of a single close encounter between planets whose small (but nonzero) eccentricities and similar semi-major axes permit such an encounter. I investigate the hypothesis that the pre-encounter orbits were nearly circular, because that is expected if planets condense from a disc. If they were not nearly circular, then the question of the origin of their observed eccentricity can only be deferred, not solved.

In §2 I present the elementary constraints implied by the conservation laws. §3 discusses the validity of the two-body approximation for the encounter between two planets orbiting a massive third body. In §4 I develop a "parabolic approximation" for such encounters, estimate the parameter regime for which it is valid. This approximation is used in §5 to constrain the parameters of planetary systems which could produce the present observed eccentric orbits after such an encounter. §6 contains a brief summary discussion.

2. Conservation Laws

The present observed planetary orbit has semi-major axis a and eccentricity e. There is no evidence for a second planet (designated 2) in any of the systems in which the observed planet has an eccentric orbit, so we assume this planet has been lost. The conservation of energy then permits constraints to be placed on the pre-encounter orbital parameters; no constraint can be obtained from the conservation of angular momentum because the angular momentum of the escaped planet is indeterminate.

If the observed eccentric orbit was produced in a single encounter, that encounter must have occurred at a distance from the star between $a(1-e)$ and $a(1+e)$, the smallest

and greatest distances of the present orbit from the star. The initial radius r_1 of the orbit of the observed planet, assumed nearly circular, satisfies

$$a(1-e) \leq r_1 \leq a(1+e). \tag{1}$$

Energy must be removed from this orbit in order to expel planet 2. The greatest amount of energy is released if $r_1 = a(1+e)$. If the initial orbit of planet 2 had semi-major axis a_2, and it is presently unbound, then the condition that the energy to unbind it came from the reduction in semi-major axis of planet 1 (from $a(1+e)$ to a) is

$$\frac{m_2}{m_1}\frac{1+e}{e}a \leq a_2, \tag{2}$$

where m_1 and m_2 are the masses of the two planets.

It is also necessary that the orbits intersect. Assuming $r_1 = a(1+e)$ yields

$$a_2(1-e_2) \leq a(1+e) \leq a_2(1+e_2). \tag{3}$$

Then

$$a\frac{1+e}{1+e_2} \leq a_2 \leq a\frac{1+e}{1-e_2}. \tag{4}$$

The right hand inequality in (4) may be combined with (2) to yield

$$1 - e\frac{m_1}{m_2} \leq e_2. \tag{5}$$

Unfortunately, without knowledge of m_1/m_2 no quantitative bound can be placed on e_2. It is expected that $m_1/m_2 \geq 1$ (the lighter planet is more likely to be lost), so that (5) may only be the trivial statement $e_2 \geq 0$.

It is possible to constrain m_1/m_2 by requiring that the impulse transferred in the encounter be sufficient to scatter planet 1 from its initial circular orbit to the observed ellipse. The exact statement of this condition is algebraically unwieldy and depends on the unknown parameters of the initial orbit of planet 2, but a simple (though not the strictest) bound may be obtained by assuming planet 2 initially in a radial orbit with zero total energy. Then the relative velocity of encounter

$$v = \left[\frac{3GM}{a(1+e)}\right]^{1/2}, \tag{6}$$

where M is the stellar mass. The momentum transfer depends on the scattering angle, but satisfies

$$\Delta p \leq \frac{2m_1 m_2 v}{m_1 + m_2}. \tag{7}$$

The momentum transfer required to scatter planet 1 into the observed orbit

$$\Delta p = \left[\frac{GM}{a(1+e)}\right]^{1/2} m_1 \left[1 - (1-e)^{1/2}\right]. \tag{8}$$

Combining these conditions yields

$$\frac{m_1}{m_2} \leq \frac{2\sqrt{3}}{1-(1-e)^{1/2}} - 1. \tag{9}$$

For $e = 0.63$ the result is $m_1/m_2 \leq 7.8$, which is not strict enough to give any useful information when substituted in (5).

In the preceding paragraph the pre-encounter eccentricity of the second planet $e_2 \to 1$. If this assumption is made the the encounter has not solved the problem of the origin of eccentricity. If, on the other hand, both planets are initially in nearly circular (but slightly eccentric and intersecting) orbits, then v is small, as must be Δp, failing to explain the large observed e. This case will be dealt with in §4.

3. Two-Body Approximation

If three bodies interact with each other simultaneously, then no simple results beyond the conservation laws apply. However, if the interaction of two of the bodies has a characteristic time short compared to the period of their motion about the third, then the interaction of the first two may be treated independent of the influence of the third. This is the case for two planets, of mass much less than that of the star, which briefly pass close to each other in their orbits around the star, as well as for the more familiar case of a short period satellite orbit or hierarchal triple star.

Considering a planetary encounter as a two-body scattering problem, define its impact parameter b and relative velocity at infinity v. Then the necessary condition for the two-body approximation is

$$\frac{b}{v} \ll \left(\frac{a^3}{GM}\right)^{1/2}, \tag{10}$$

where a is now the distance of the encounter from the star (approximately equal to the semi-major axes of the planetary orbits). For planets in orbits of slightly different semi-major axes or small but finite eccentricities

$$v \approx \frac{1}{2}\frac{\delta a}{a}\left(\frac{GM}{a}\right)^{1/2}; \tag{11}$$

where $\delta a \equiv \max(|a_1 - a_2|, e_+ a, ia)$, $e_+ \equiv e_1 + e_2$ and i is the relative inclination. Henceforth I will assume the orbits to be coplanar and $\delta a \ll a$. Then the necessary condition (10) becomes

$$b \ll \delta a. \tag{12}$$

This condition can be met if the orbits intersect, as is possible if $e_+ a$ is greater than the difference in semi-major axes, and is implicitly assumed by the hypothesis that observed eccentric orbits are produced by close encounters. It is consistent with small pre-encounter eccentricities, provided that the initial semi-major axes are similar enough. This is not observed for planets in the Solar System, but there is no problem of planetary eccentricity to explain there. It is found for small bodies such as asteroids.

4. Parabolic Approximation

In two-body scattering problems it is frequently possible to make an impulsive approximation, corresponding to small deflections, in which the momentum transfer is computed as if the initial trajectories of the particles continued undeflected through their interaction. In the present problem the opposite "parabolic approximation" can be made, in which the path of their separation vector is nearly parabolic, and their relative velocities change sign as a result of their interaction. The momentum transfer is then independent of b even if $b \to 0$ and the particles approach arbitrarily closely and feel arbitrarily strong forces (in the planetary problem it is necessary to exclude the extreme case of actual impact). The parabolic approximation is valid for impact parameters

$$b \ll \frac{Gm}{v^2}, \tag{13}$$

where $m = m_1 + m_2$.

Planets in nearly circular orbits with semi-major axes differing by $\sim \delta a$ will have relative velocities

$$v \sim \left(\frac{GM}{a^3}\right)^{1/2} \delta a. \tag{14}$$

Combining (13) and (14) and taking $b \sim \delta a$ yields a sufficient condition for the validity of the parabolic approximation (it is, of course, valid for smaller b):

$$\frac{\delta a}{a} \ll \left(\frac{m}{M}\right)^{1/3} \approx 0.1, \tag{15}$$

where the numerical value is appropriate to Jupiter-like planets orbiting Solar type stars. The parabolic approximation is valid for all encounters (which may be defined as the overtaking of the longitude of one planet by that of the other) between planets whose semi-major axes satisfy (15). It is also valid for encounters with $b \ll ma^3/M\delta a^2$) between planets with larger δa (this may be considered the definition of intersecting orbits).

Equation (15) must hold if there are to be close encounters between planets in nearly circular ($e_+ < (m/M)^{1/3} \approx 0.1$) orbits. Hence the parabolic approximation is always applicable to the problem discussed in this paper. If it is not valid then at least one of the initial orbits was significantly eccentric, and the encounter has not produced an eccentric orbit from nearly circular orbits.

5. Results

The parabolic approximation simplifies the dynamics, because in a parabolic encounter the planetary velocities (in the frame of their center of mass) simply reverse sign. It is then possible to relate the observed eccentricity to the pre-encounter eccentricity e_2 of planet 2, assuming planet 1 was initially in an exactly circular orbit of radius $a(1 + e)$. The assumption of small eccentricities implies that the pre-encounter velocity of planet 2 was nearly circumferential. The requirement that planet 2 be expelled is most easily met

if $a(1+e) = a_2(1-e_2)$, in which case both velocities are exactly circumferential. The required momentum transfer is given by Equation (8). The relative velocity is then

$$v = \frac{m_1 + m_2}{2m_2} \left[\frac{GM}{a(1+e)}\right]^{1/2} u, \tag{16}$$

where $u \equiv 1 - (1-e)^{1/2}$. Using the pre-encounter circular Keplerian velocity of planet 1, the pre-encounter velocity of planet 2 may be calculated, and from that its total energy:

$$E_2 = \frac{GMm_2}{a(1+e)} \left(-\frac{1}{2} - \frac{u}{2\mu} + \frac{u^2}{8\mu^2}\right), \tag{17}$$

where the mass fraction $\mu \equiv m_2/(m_1 + m_2) < 1$. Similarly, the pre-encounter angular momentum of planet 2 is

$$L_2 = \left[GMa(1+e)\right]^{1/2} m_2 \left(1 - \frac{u}{2\mu}\right). \tag{18}$$

From these quantities its eccentricity is obtained (Symon 1960):

$$e_2 = \left(4u^2 - 4u^3 + u^4\right)^{1/2}. \tag{19}$$

In the limit $e \rightarrow 0$

$$e_2 \rightarrow \frac{e}{2\mu}. \tag{20}$$

In order to evaluate Equations (19) or (20) explicitly it is necessary to know μ. For $m_1 = m_2$ ($\mu = 1/2$) Equation (19) reduces to $e_2 = e$; the planets simply exchange orbits. This neither solves the eccentricity problem nor is consistent with the expulsion of planet 2. If $m_1 > m_2$ then $e_2 > e$ so that a larger eccentricity is required before the encounter than is found after it. If $m_1 < m_2$ then $e_2 > e/2$; the pre-encounter eccentricity may be smaller than that observed, but by no more than a factor of two.

The condition (5) on m_1/m_2 must be satisfied in order than planet 2 be lost. From this condition it is possible to calculate (iteratively) a lower bound on e_2 as a function of e. The results are shown in Figure 1. For $e < 0.5$ the minimum e_2 ($e_2 > e$) is found for $m_1 > m_2$. For $e > 0.5$ solutions are possible with $e_2 < e$; these solutions have $m_1 < m_2$, but large e_2. If $m_1 \geq m_2$ is required then $e_2 \geq e$. In computing Figure 1 the parabolic assumption was made, which is not valid for all encounters permitted when the eccentricity is large. However, non-parabolic encounters transfer less momentum than parabolic ones (the scattering angle is less than 180°), and therefore require greater relative velocity and larger pre-encounter eccentricity than the assumed parabolic encounters.

6. Discussion

It is not possible to produce the observed eccentric orbits by single encounters between planets initially in nearly circular orbits. This conclusion is almost obvious: scattering

from a circular orbit to one with substantial eccentricity requires substantial momentum transfer, and hence a substantial relative velocity between the two planets, which cannot be obtained if they are are initially in coplanar and nearly circular orbits of nearly the same size.

How were the observed eccentricities produced? It is not possible to make any general arguments concerning the effects of small perturbations acting over the age of the planetary system, or constraining the effects of possible repeated close encounters. Mazeh and Krymolowski (1996) have suggested that the eccentricity of the planet orbiting 16 Cyg B may be attributable to the presence of the distant companion star 16 Cyg A in an (assumed) inclined orbit, while Rasio and Ford (1996) have suggested that single planets in tightly bound orbits could result from the long term interaction of planets in orbits initially satisfying Eq. (15). These numerical results illustrate the potential richness of orbital evolution produced by the integration of small forces over long times. Certainly (and fortunately) the major planets do not induce such large eccentricities in Earth's orbit, or their own.

The possibility that a mechanical collision between two planets could produce an eccentric orbit may also be dismissed. Such a collision between Jupiter-like gaseous planets would be dissipative, and would have the effect of averaging their orbital parameters. If their initial orbits were nearly coplanar and nearly circular the result could only be a nearly circular orbit.

It may be useful to consider the early history of the eccentric planetary systems. For example, tidal interaction with a rapidly spinning star might make the eccentricity of a planetary orbit grow, as the accelerating tidal torque is largest at periastron. Tidal interaction is negligible when the star is of Solar dimensions and a is an AU or more, but may have been significant during the pre-main sequence or protostellar stages of evolution, when the star was larger. Eccentricity induced by a rapidly rotating star has a characteristic signature: close planets have larger eccentricities than more distant ones. The applicability of this process to systems like 16 Cyg B would be tested if additional more distant planets are found accompanying the observed eccentric planets, and it is consistent with the absence of planets in orbits inside the observed eccentric orbits. It might explain the comparatively large ($e = 0.206$) eccentricity of Mercury.

I thank T. Mazeh and T. Piran for discussions, NSF AST 94-16904 and NASA NAG 52682 for support, the Hebrew University for hospitality and a Forchheimer Fellowship, and Washington University for the grant of sabbatical leave.

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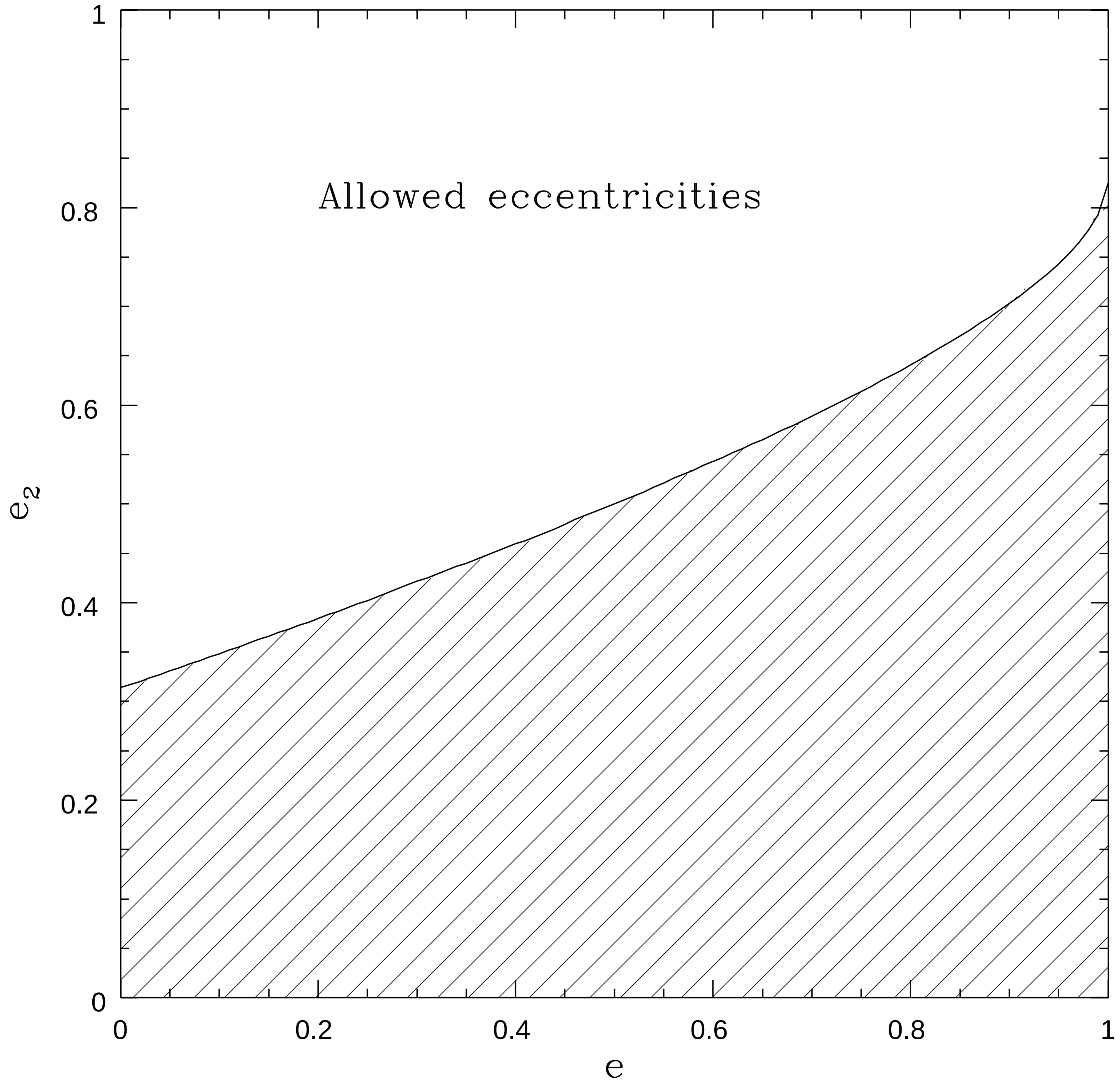
Figure Caption

Figure 1: Allowed initial eccentricity e_2 of unseen planet 2 as a function of observed eccentricity e. The planetary mass ratio m_1/m_2 has been found implicitly, assuming that planet 2 was expelled as a result of the encounter. The parabolic approximation was made, and planet 1 was assumed initially in a circular orbit of radius $a(1+e)$.



Allowed eccentricities
e_2
e
0
0.2
0.4
0.6
0.8
1